OPTIONS RELATIONSHIPS

Understanding options relationships is useful because:

1. It solidifies your understanding of what options do;
2. It might suggest trading/investing strategies driven by these relationships, known as “relative value trades;
3. It provides a gateway to valuing an option; i.e., to, finally, examining the determinants of the premium.

Our focus in this and succeeding chapters is no longer limited to options at their expiration. The relationships analyzed in this chapter are between an option and: a) a dollar value; b) the underlying asset; c) another option. We will derive most of them through arbitrage arguments, but we will at times rely on present value/probabilistic reasoning. A crucial concept is “fair forward value.” It is fundamental to all derivatives, and necessary for our discussion here (as well as later chapters). We introduce it after we get through some simple relationships in the next section.

**OPTIONS VALUES ARE NEVER NEGATIVE**

The idea that option premiums cannot be below 0 is almost self-evident. [[1]](#footnote-1) Still, it is worthwhile proving it in order to introduce the method of arbitrage in the simplest case.

Suppose shares of J.W. Nordstrom (JWN), an operator of department stores, are at 72. A call with an exercise price of 75 is priced at −1. A price of −1 means the owner of the call is willing to pay 1/share in order to relieve himself of it. AN arbitrageur would do the following:

1. Take (i.e., “buy”) the call from the owner and receive 1.
2. Hold on to the call until expiration (or possibly exercise).

The worst case scenario is JWN never rises above 75. In that case, the arbitrageur nets 1/share. If JWN does rise above 75, exercising the option will produce even greater profit. In short, “paying” −1 for the call produces a riskless profit of at least $1, and possibly more. All rational market participants will be clamoring to do this. They will besiege the call holder, who will have an incentive to change his asking price from −1 to, say, −0.5. But this does not erase the arbitrage. The process continues until the price is at least 0. We can write this as Call75 >0. As this argument will hold for any exercise price, K, we can write:

CallK ≥0 for all K.

A similar argument can be made for a put. Suppose a put on Nordstrom struck at 70 is priced at −0.5/share. Buying the put means receiving 0.5, plus the right to sell JWN for 70. If JWN never falls below 70 the arbitrageur nets 0.5. In addition, she might do better, if JWN drops below 70. Because a profit of at least 0.5 is assured, investors will push the price of the put to at least 0. In other words:

PutK ≥0 for all K.

Another way to describe these relationships is by saying that the *minimum value* of either a cal or a put is zero:

min(Call) = 0

min(Put) = 0

Of course, from the writer’s perspective, the option has a value less than 0 (exactly equal , in absolute value, to the positive value for the option buyer). When we speak of an option’s value – minimum or otherwise – we speak from the perspective of the owner, just like the owner of any other item.

**EXPECTED VALUE**

The value of an asset – be it a stock, a bond, real estate or an option – is the present discounted value of the sum of its expected future net cash flows. The expected future cash flows of a call option equal the probability associated with hitting each price above the strike multiplied by that price.[[2]](#footnote-2) As long as there is any probability at all of Nordstrom stock hitting a price above 75 at the call’s expiration, the call is worth something greater than 0. For a put, the expected future cash flows reflect probabilities below the strike. [[3]](#footnote-3) As long as there is any chance of JWN falling below 75, the put is worth more than 0. (The discounting factor, reflecting the risk of the outcomes, reduces the present values of the call and put. But it cannot bring the values below 0.) We have, therefore:

CallK > 0 for all K

PutK > 0 for all K

Note there is no arbitrage that *assures* the strict inequalities hold. They hold only on an expected valuation basis.

**RELATIVE PRICES OF CALLS**

In chapter III we used arbitrage arguments to prove that: K1

CallK1 ≥ CallK2  for K2 > K1

We then employed the principle of dominance, which is really a variant of the present value of future expected cash flows, to conclude:

CallK1 > CallK2  for K2 > K1

Combining, we have:

CallK1 > CallK2  > 0 [[4]](#footnote-4)

In the same way, chapter III showed that arbitrage results in:

PutK2 ≥ PutK1  for K2 > K1

And the dominance principle , which is really a variant of the present value of future expected cash flows, produces:

PutK2 > PutK1  for K2 > K1

**Maximum Values of Options**

A call’s premium can never exceed the price of its underlying asset. Suppose, with FaceBook shares priced at 75, a call struck at 50 trades at 80. Sell the call and purchase the stock for a net gain of 80−75=5. If FB remains below 50 at expiration, you can sell it because the call expires worthless. This adds to your profit. Even if FaceBook falls to 0, you keep at least the 5. If FaceBook passes 50 and the call is exercised, you simply part with the stock you own and receive 50. Since this is a no-lose strategy, arbitrageurs will sell the call and buy the stock, pushing down the price of the former and bidding up the price of the latter, until the call premium no longer exceeds the cash price. Hence:

CallK ≤ S for all K, S

Where S is the spot, or cash, price of the underlying.[[5]](#footnote-5) Together with the earlier result:

0 ≤ CallK ≤ S

A put’s premium can never exceed its strike price. Proving this via arbitrage is left as an exercise. Therefore:

0 ≤ PutK ≤ K

European vs. American

European options, as introduced in chapter 1, are exercisable only at expiration.[[6]](#footnote-6) An American style may be exercised any time prior to expiration.[[7]](#footnote-7) Consider an American and a European call or put on the same underlying asset and with the same strike price and expiration dates. All the rights conferred upon (and paid for by) the European option holder apply to the American holder as well, plus more. The American option, therefore, can never be worth less than its European counterpart:[[8]](#footnote-8)

American Call ≥ European Call

American Put ≥ European Put

**Intrinsic Value**

Suppose JWNordstorm shares are trading at $72. A three-month call has an exercise price of $70. It is said to have an *Intrinsic Value* of 2 dollars a share.

An option’s Intrinsic Value is the option’s payoff at the current price

of the underlying asset.

Were JWN to remain at its current price through expiration (or, were it to fluctuate yet close at 72 on the expiration date), the call would be worth $2. This is its Intrinsic Value (IV).[[9]](#footnote-9) If, instead, JWN is trading at 80/share in the spot market, the call’s IV is 8/share. If JWN is 65 in the cash market, the call’s Intrinsic Value is *not* −5. It is, rather, 0, because at expiration a price of 65 produces a payoff of 0. On the other hand, a different call on JWN, struck at 60, would have an IV of +5 with JWN at 65. Thus:

Intrinsic Value Call = max [0, S ─ K]

The IV definition for a put option is the same. With JWN at 72, a put struck at 75 has an Intrinsic Value of 3. Were JWN to remain at 72/share at expiration, the put buyer would earn 3/share by delivering to the put writer for 72/share. If JWN is priced at 70 in the cash market, the Intrinsic Value is 0. Therefore:

Intrinsic Value Put = max [0, K ─ S]

An equivalent definition for Intrinsic Value is the following: the amount of money to be received now were the option to be exercised. Obviously, it would only be exercised if it is in-the-money, with the Intrinsic Value equal to the amount it is ITM. Note that this definition applies, as does the above, to European as well as American style options because it is a conditional statement. It is more appropriate, however, for American options since the intrinsic value can actually be realized at any time. So, for American options, and for American style options only, the call or put must be worth at least its Intrinsic Value:

American Call ≥ I.V.

American Put ≥ I.V.

Although the Intrinsic Value of a European option cannot be realized prior to expiration, it is common practice to describe a European option that is in-the-money as having Intrinsic Value. The reasoning is this: nothing needs to change in order for the option to produce income at expiration equal to its Intrinsic Value.[[10]](#footnote-10) How – and whether – IV enters the actual market value of the option, its premium, is explored in subsequent sections.

FAIR FORWARD VALUE

We come now to a concept central to a deeper understanding of options. This is the concept of *fair forward*. Fair forward will also be useful for a correct appreciation of the value, or price, of an option that we will explore in subsequent chapters.

Every asset has a fair forward value. A forward contract is an agreement today between a buyer and seller to transact on a future date. Crucially, the price of the transaction is set today.[[11]](#footnote-11) The price is “fair” when it presents no arbitrage opportunities; i.e., it is not unfair relative to the price of the underlying asset in the cash market and other market variables. Let’s apply this concept to gold. Gold is a good place to begin because it obviously pays no dividends. Suppose the price of gold is $1200 per ounce in the spot market. Suppose further that a potential counterparty exists who is prepared to enter a contract with you, or anyone, to purchase gold for $1260/oz in one year. Assume the one-year interest rate is 4%. A riskless arbitrage has presented itself to you:

1. Enter a one-year forward contract with the counterparty to sell gold to him for $1260/oz.
2. Borrow $1200 for one year at 4%;
3. Use the money purchase the gold in the spot market (convey the gold to the lender as collateral);
4. At the end of the year repay the lender $1200 plus 4%, or $1248 (retrieve the collateral).
5. Deliver the gold to the counterparty in satisfaction of the contract and receive $1260.

You have made a profit of $1260 (revenue from step 5) less $1248 (cost from step 4) = $12 with no risk and no cash outlay on your part. No risk because, as is clear from the example, the price of gold in the cash market in one year has no bearing on the dynamics. Your profit of $12 occurs no matter what the price of gold then.[[12]](#footnote-12) Every rational market participant will be attracted to the same arbitrage – free money! They will bombard the forward contract counter-party with offers to sell him gold in one year (and, simultaneously, enter the cash gold market with bids to purchase gold) at his price. This will force him to lower the price he is willing to pay (and raise the price of spot gold). The process will not end before the forward contract price equilibrates at $1248/oz (or the spot rises, or some combination of the forward contract price falling and the spot price rising until arbitrage is erased).

What we have proved is that the one-year forward price cannot be above $1248/oz. Can it be below $1248? No. Suppose it is. Say a potential counterparty is willing to sell gold in one year for $1240/oz. AN arbitrage exists again:

1. Enter a one-year forward contract with the counterparty to *buy* gold from her for $1240/oz.
2. Borrow gold for one year;
3. Sell the gold in the spot market and lend the cash at 4% (or convey the cash to the lender as collateral who will pay 4% interest);
4. At the end of the year deliver $1240 to the counterparty in satisfaction of the contract and receive gold.
5. Return the gold to the lender and receive $1200 plus 4%, or $1248 from the loan (or from the gold lender who had the cash collateral).

Again you have effected a riskless profit, this time $8, in one year regardless of the price of gold then.[[13]](#footnote-13) Everyone will do this, forcing the contract price up (and the spot price down), until the arbitrage is

erased. We have proved that, with spot gold at $1240 and the one-year interest rate at 4%, no one would be willing to enter a one-year forward contract to sell gold for less than $1248/oz, and no one would be willing to pay more than $1248. In short, no transactions can take place except *at* a price of $1248 – the fair forward price.

Algebraically, the first arbitrage tells us that the forward price of gold cannot exceed the cash price plus the cost of financing the gold (the interest expense for borrowing the purchase price) until the forward contract settlement date, or:

Forward ≤ S x (1 + r x t) ,

where r is the financing, or interest, rate on the borrowing) and t is the time until the contract’s settlement.[[14]](#footnote-14) Otherwise, riskless profits may be earned by purchasing the gold, financing it and selling the gold forward at the contract price. The second arbitrage tells us that the forward price of gold cannot be below the cash price plus the earnings from selling short gold (the interest received for investing the sale price) until the forward contract settlement date, or:

Forward ≥ S x (1 + r x t) .

Otherwise, riskless profits may be earned by borrowing and selling short gold, investing the proceeds and purchasing the gold forward at the contract price to cover the short. Since both inequalities must be simultaneously satisfied, we must have the strict equality:

FF = S x (1 + r x t) ,

where FF denotes the fair forward price of the security, S. It is “fair” in the sense that it is the only price for a forward contract setting t years from now that precludes arbitrage profits.[[15]](#footnote-15)

**Implications**

A number of important implications arise from the Fair Forward price concept/equation:

* In general, the longer the time to settlement, the higher the fair forward price. For example, the one-year fair forward gold price, with spot gold at $1,200 an ounce and the one year interest rate 4%, is $1248, as concluded above. The six-month forward price is 1200 + (.04/2)x1200 = 1224[[16]](#footnote-16) It is possible, though uncommon, for shorter-dated forward prices to exceed longer-dated prices. This would occur in our example if the six-month interest rates were significantly higher than one-year rates; i.e., an invested yield curve.
* There is one spot price for gold. But there are theoretically an infinite number of forward prices, depending on settlement dates.[[17]](#footnote-17) In practice, liquid contracts (narrow bid-offer spreads) are limited to common settlement dates, such as one month, three months, six months, etc.
* For a given spot price, forward prices approach, or “converge,” to the spot price over time. For example, the one-year fair forward is $1248, as above. Assume that six months later spot gold remains at $1200. The “seller” of the contract – the counterparty who agreed to sell at the forward contract price of 1248 – has made a profit. Why? Because the original one-year contract has become a six-month contract, and the six-month fair forward price is now 1224 (assuming unchanged interest rates). All she needs to do is enter an offsetting six-month contract as a buyer, and her profit of 24/oz is locked in. By the same token, the buyer” of the contract – the counterparty who had originally agreed to pay 1248 in one year – has lost 24/oz. Of course, this is all based on the assumption of no change in the spot price of gold between the contract date and six months later.[[18]](#footnote-18)
* At any point in time, a change in the spot price is mimicked by a change in the forward price. If not, Indeed, the reverse is also true, as it is immaterial if the change originated in the spot or forward market. This assumes stability in the financing rate. The implication is clear: a speculator on the price of gold does not need to take a position in cash gold. A forward contract will present the same exposure.

The most important implications of an asset’s fair forward price for options involves the option’s minimum value. Before we explore that, we need to look at fair forward prices for assets other than gold.

**Income Producing Assets**

Consider Ford Motor (F) shares priced at 15 in the cash market. Ford pays an annual dividend of 0.5/share. Suppose a counterparty exists who is willing to pay 15.25 per share for Ford in one year. Take him up on it and make a riskless profit! Go through the same steps as in the first panel above, substituting Ford for gold as well as the respective prices. Briefly, borrow $15 for each share you purchase and, simultaneously, enter the contract to sell at $15.25. You’ll owe $15.60 at year’s end, but you’ll be receiving $15.25 *plus the 0.50 dividend*.[[19]](#footnote-19) You’ve made a profit of .15 per each Ford share, with no risk (and no money out of pocket). This cannot last nor, indeed, can it exist. It must be the case, therefore, that the Revenue is less than the cost, or:

Forward + div ≤ S x (1 + r) ,

where div is the annual dividend per share. To generalize, let Forwardt denote the forward price for a contract settling t years in the future, and rewrite as:

Forwardt ≤ S x (1 + r x t) ─ div x t = S x [1 + (r ─ div/S) x t) .

In our example t=1, so that the forward price must be no greater than 15.10 per share. The dividend per share as a percentage of the share price – div/S – is known as the dividend “yield.”

Suppose for a moment the one-year forward contract price in the market is 15. Duplicate the second set of steps above, properly substituting Ford and its prices for gold. You sell Ford short by borrowing the stock and earn the interest rate on the proceeds. You will be liable for the dividends while you are short, so you and all the others who see this opportunity will force the cost to exceed the revenue:

Forward + div ≥ S x (1 + r) .

Or, more generally:

Forwardt ≥ S x [1 + (r ─ div/S) x t) .

Because both inequalities are operative, it must be the case that:

FFt = S x [1 + (r ─ div/S) x t) .

In our example, the only fair price for a one-year forward contract on Ford shares is 15 x [1 + .04 ─ .5/15) = 15.10.

The set of four implications above for the Fair Forward price of gold applies to the Fair Forward price of a stock, S. The only change is r-div in place of r. When the interest rate exceeds the stock’s dividend yield, the fair forward price exceeds the spot price. And the fair forward is higher the longer time to the forward date.[[20]](#footnote-20) When the interest rate is below the dividend yield, the fair forward is below the spot. Again, the difference widens with time to settlement. A stock’s dividend yield is its “carry.” In general, a security’s carry is its earnings, quoted as a percentage, assuming no change in its price. “Net carry” equals carry less the cost of financing because that is your net earnings, assuming you’re borrowing money to buy the security, simply due to the passage of time. Another war to say this is that r is the cost of carry, so that r less div is the “net cost of carry.” We have, therefore:

= =

The difference between the forward and spot price, as a percentage of spot, is a direct function of the net cost of carry of the asset.

* The dividend yield of common stocks generally are lower than short-team interest rates, hence financing costs. , therefore, is positive, and increases with time to settlement.[[21]](#footnote-21)
* Real Estate Investment Trusts (REITS) must pay out at least ninety percent of their income in the form of dividends. Their fair forward prices will be below spot, and decrease with time o settlement.
* The relationship between a bond’s yield and the cost of financing a bond will depend on the shape of the yield curve. The more steeply sloped the curve and, therefore, the longer the maturity of the bond, the greater the excess of the bond’s yield over its financing cost.[[22]](#footnote-22) Hence, the deeper the discount of its forward rice relative to spot.
* Gold, of course, pays no dividend. Its net cost of carry is simply r, so that its forward is always at a premium to spot.[[23]](#footnote-23)
* The relationship between forward and spot currency exchange rates depends on the difference between the short-term interest rates of the relevant currencies. The home currency rate and the foreign currency rate replace r and div/y, respectively, in the equation.

[ Storage and related costs for physical gold can be avoided by employing an ETF. ]

Minimum Value of Option

This is all interesting. But what has it to do with options? Quite a lot, it turns out, and in a fundamental way.

We concluded above that the one-year fair forward price of Ford stock is $15.10. Consider a one-year European call on Ford with an exercise price of 14. Purchasing the option allows you to pay $14 in one year for something that the market deems worth $15.10. You are, therefore, earning, or saving, $1.10. But because that will not be paid to you until one year from today, its present value is $1.10/1.04 = $1.06. The call option, in other words, is worth *at least* (15.10 – 14)/1.04 = 1.06.

We can arrive at this result via an arbitrage. Because Ford’s fair forward price is $15.10, you can enter a contract to sell a share a share in one year at that price. Simultaneously, purchase a one-year call struck at $14. If Ford is trading at or above 14/share in one year, exercise the call and the deliver on the contract to make a net profit of $1.10. If Ford is trading below 14/share, say 13.50, discard the call and purchase Ford in the spot market then to make a net profit of $1.60. You’ll always make *at least* $1.10 in one year. Market participants will, therefore, bid up the price of the call to at least the present discounted value of $1.10.[[24]](#footnote-24) We conclude that the arbitrage value of the call *today* is 1.1/1.04 = 1.06. By extension, the option must be worth this value, at a minimum.

Here is a profound implication. Were this an American option, exercising it now would produce a profit of $15−14=1.00. Yet, we just proved that the option is worth at least $1.06. Why would you exercise to earn $1.00 when you can sell it for at least $1.06?! Early exercise, in this case, is irrational. In other words, the “American” feature adds no value, and no one would pay extra for it.

We can generalize this observation. For non-dividend paying stocks, as well as for gold and other commodities, the fair forward price exceeds the spot.[[25]](#footnote-25) A call’s arbitrage value in these cases exceeds its intrinsic value. The benefit of early exercise is absent. Hence, an American style option is worth no more than a European.

Consider now the same option struck at 15. Employing either of the above arguments, its arbitrage value is (15.10 – 15)/1.04 = 0.10. Although the call is at-the-money, so that is has zero intrinsic value, it has a minimum value greater than zero!

What about an exercise price of 16? Giving the call holder the right to pay 16 in one year for something worth 15.10 provides no arbitrage. The arbitrage value is *not* a negative 0.90. In this case it is simply 0. In general, therefore:

Arbitrage Value Call = max{0,(FF − K)/(1+rxt) } ,

with FF, the fair forward value, as given above. We conclude:

The minimum value of a call option – *European or American* – is its Arbitrage Value.

Let’s consider some cases where early exercise may be economical. Verizon Communications (VZ) pays a dividend of $2.20 per share. At a price of 48/share its dividend yield is 4.58%. Its six-month fair forward price is 48x[1+(.04−.0458)/2] = 47.86. A six-month call at-the-money has no arbitrage value nor intrinsic value. A call struck at 47, on the other hand, contains arbitrage value equal to (47.86-47)/1.04 = 0.83. However, it has intrinsic value of (48-46) = 1, which makes early exercise a valuable option. The minimum value of an American option, therefore, is 1.00, but that of a European option is 0.83. We conclude:

Min value American call = max{0, Arb Value, Intrinsic Value}

Recognize that, just as American-style call options, European calls have intrinsic value when the spot price of the underlying exceeds the option’s strike. But their intrinsic value cannot be *realized* (before expiration). It does not, therefore, enter the European minimum value.

Puts

Let’s go through the same process with puts. Given Ford’s dividend yield of 10 cents per share and the relevant financing rate of 4%, the one-year fair forward price is $15.10 compared to its cash rice of $15. A put with a one-year expiry and struck at 14, therefore, has no arbitrage value – it doesn’t allow you to lock in anything that the market doesn’t already give you. In fact, raising the strike to 15 produces no arbitrage value. Unlike the 15-call, which lets you buy Ford in one-year for 0.10 less than the market does, the put allows you to sell at 15 whereas the market will give you 15.10. A 16 strike, though, exceeds the fair forward. Hence a put’s arbitrage value equals (16-15.1/1.04 = 0.87. More generally:

Arbitrage Value Put = max{0,(K − FF)/(1+rxt)} ,

which becomes a (European or American) put’s minimum value.

The arbitrage argument employed above for calls as applicable here as well. You can enter a contract today to sell Ford in one year for 16/share. Do so and buy the put today. You have locked in a sale price of at least 16/share. And, if Ford is above 16 in one year, you will not exercise, rather sell in the spot market then. You have, therefore, a certain profit of 0.90, which means the put’s premium must be at least 0.90/1.04 = 0.87 to preclude arbitrage.[[26]](#footnote-26)

What about a put’s intrinsic value as it relates to its minimum value? With Ford at 15, a put struck at 14 or 15 has no intrinsic value (just as it has no arbitrage value). But a 16 strike put has an I.V. of 16−15 =1. This exceeds its arbitrage value of 0.87. Early exercise of the put *is* a rational decision.[[27]](#footnote-27) The minimum value of an American Put, in this case, exceeds that of a European. Or, in genera:

Min value American put = max{0, (K − FF)/(1+rxt), K − S}

Exercises

Stock ABC is priced at 40. The relevant financing rate is 5%. For each of the following examples, calculate:

fair forward price; arbitrage and intrinsic values.

Then:

minimum values of American and European calls and puts,

given the various parameters and assumptions. Follow the format shown in example 1.

1. Assumptions: dividend = 1.25/share; six-month option; exercise price = 39

fair forward = 40 x (1 + .05/2) − 1.25/2 = 40.38

call: arb value = (40.38 – 39) (1 +.05/2) = 1.35; intrinsic value = 40 – 39 = 1

Therefore, min value of European = American = 1.35

put: arb value = 0; intrinsic value = 0

Therefore, min value of European = American = 0

1. Assumptions: dividend = 1.25/share; one-year option; exercise price = 41
2. Assumptions: dividend = 2.25/share; two-year option; exercise price = 41
3. Assumptions: dividend = 0; six-month option; exercise price = 42
4. Assumptions: dividend = 2.25/share; one-year option; exercise price = 38
5. Assumptions: dividend = 2.25/share; one-year option; exercise price = 39

PUT-CALL PARITY

Our final options relationship is different from the above, as it relates a call’s value to that of a put (as opposed to the relationship between two calls, or puts, or between an option and a number or an option and a forward price). Further, it will be crucial for concepts in later chapters.[[28]](#footnote-28)

FaceBook is 75/share. It pays no divided, and it costs 4% to borrow money in order to purchase shares. Consider a one-year cal and a one-year put, both European and struck at 80. Perform the following transactions:

* Borrow 75 and buy the stock.
* Write the call and purchase the put.

The following will happen in one year:

* You owe 75x1.04 = 78 to the lender of funds.
* If FB is below 80, you will exercise the put – you will receive 80 and deliver FaceBook to the put writer; the call expires.
* If FB is above 80, the buyer of the call will exercise – you will receive 80 and part with FaceBook; the put expires.
* If FB is exactly 80, you will either sell FaceBook in the market and allow your put to expire, or exercise your put (or the call owner exercises). In any case, you receive 80.

In short, you have locked in a sale price of 80 in one year. This means that, together with the 78 you owe in one year, you have a certain profit of 2 per share of FB. Arbitrageurs, therefore, will force the put-call combination today to have a price of 2 discounted by the 4%. We have shown that:

This enormously important equation tells us the following: Given the price of the underlying asset, the relevant interest rate and time to expiration, then for any put-call combination of the same strike, knowledge of the premium of one forces the premium of the other. In other words, calls and puts with common exercise prices cannot be priced independently.[[29]](#footnote-29) In our case, if the call is trading at 1, the put must be worth (80−75x1.04)/1.04 + 1 = 2.92. A premium below 2.92 for the put would invite the above arbitrage. What about a premium below 2.92? We’ll deal with that in a moment.

It is instructive to arrive at the same result via forwards, using the arbitrage argument of the previous section which established minimum values. With no dividend and a 4% financing rate, FaceBook’s one-year fair forward price is 78/share. Enter a forward contract to purchase FB and simultaneously buy the 80- put and write the 80-call. You have effectively purchased FaceBook in one year for 78 and sold it hen for 80. The put-call pair must cost you 2/1.04 = 1.92. Or:

Let’s review. Suppose purchasing the put and writing the call costs less than net 1.92, say 1.80. Doing so and either: buying FaceBook today and financing it for a year; or, buying it forward in one-year, will produce a risk-free profit. Revenue from the sale via the option pair is 80 while the cost – either the borrowing or the forward contract – will be 78. The 1.80 you spent today becomes 1.80x1.04 = 1.87 in one year. Your profit is 2 – 1.87. Arbitrageurs will buy the put and write the call, putting upward pressure on the put premium and downward pressure on the call price until the net expense is 1.92. At this point there is no arbitrage because 1.92x1.04 = 2.

Suppose the premium of the put less that of the call exceeds 1.92, say 2.05. In that case you should buy the call and sell the put, which will put $2.05 in your pocket. Regardless of the price of FB in one year, you will be paying 80 for it and receiving the shares. Borrow the shares today for a year and sell them short for 75/share. Or, sell them forward for 78/share in one year. In the first alternative you will invest the 75 proceeds from the short sale, which produces revenue of 75x1.04 = 78 in one year. And you will need to cover the short. In the second, you will receive 78 from the forward contract counterparty, to whom you must deliver the shares. In either case, you acquire the shares by way of the options, paying 80, and use the shares to cover your short or fulfill your forward contract. This produces a net of −2. But you have 2.05x1.04 from the net premiums (plus interest), or a riskless profit of 0.13. Arbitrageurs will do as you did, pushing down the price of the put and forcing up the premium of the call. This will continue until the net premium is 1.92. In equilibrium, therefore, we must have:

the same result as above.

If the underlying stock pays a dividend, the same arbitrage applies. The dividend received for a long position in the stock subtracts from the carry cost, and the dividend that must be paid for a short position subtracts from the earnings on the cash received. The fair forward recognizes the dividend factor, as shown earlier. The put-call parity relationship then becomes:

Here’s an immediate result of the P-C parity condition.

If and only if the options are struck at-the-money forward – i.e., K = FF – the call and the put have equal values.

If the forward price exceeds the exercise price, the call’s premium exceeds that of the put. Conversely for an exercise price above the forward. In the former case, the call has arbitrage value; in the latter it is the put. If the options are struck at-the-money spot – K = S – then the call and put will have unequal values (unless, of course, FF = S). The relationship between the two premiums depends on the relationship between the spot and forward prices: FF > S produces Call > Put, and vice-versa.[[30]](#footnote-30)

RELATIONSHIP BETWEEN BINARY PUT AND CALL; MAX PRICE OF BINARIES

[OPTIONS ON FUTURES ]

BUTTERFLIES GET A SEPARATE CHAPTER

END-OF-CHAPTER QUESTIONS

I Show that if a put’s premium exceeds its exercise price, arbitrage ensures a profit of at least the difference between the exercise price and the premium.

II Explain intuitively why, for a non-dividend paying stock (or gold), the minimum value of an ATM call exceeds that of an ATM put.

III Consider shares of Verizon as described in the previous section. Show that given those parameters, a one-year put struck at 50 has both intrinsic and arbitrage value, but early exercise is irrational, hence an American is worth no more than a European. What might change during the life of the option that may make early exercise rational? What does that do to the difference between an American and European put?

IV Consider a call option on XYZ struck at 35, with XYZ at 40 in the spot market. XYZ pays no dividend. The call has a knock-out barrier of 42. What is the relationship between the minimum values of an Ameliorant vs. a European?

V Consider a 1-year ITM-forward call option on XYZ company shares.

a) Which costs more, the call or a put with the same strike?

b) Suppose during the trading day the call premium increases without any change in the underlying spot price, interest rates or dividend policy. What must happen to the price of the associated put?

c) Suppose during the trading day XYZ increases and the call premium increases by precisely the same amount (no change in interest rates or dividend policy). What must happen to the price of the associated put?

d) Assume instead the call is OTM-forward. Does your answer to any of the above change?

VI Consider 1-year ITM Binary Call. Assume dividend yield equals interest rate so that forward and spot are the same. Which costs more, the call or a put with the same strike?

1. As explained in the opening section of this book, derivatives other than options have a net present value equal to zero at inception. Over time, in response to changes in relevant market circumstances, the derivative will move to a positive value for one counterparty, and an equal, but opposite, negative value for the other. AN option, by contrast, will have a value different from 0 even without any market changes. [↑](#footnote-ref-1)
2. Or, , where S is the price of the undleying asset, in this case JWN shares, Prob(S) is the probability function and K is the strike, in our case 75. [↑](#footnote-ref-2)
3. Reverse the K, points in the integral of the prevous footnote. [↑](#footnote-ref-3)
4. By extension, CallK1 > CallK2  > CallK3  > 0 for K3 > K2 > K1. This leads to: limK→∞(CallK)=0. [↑](#footnote-ref-4)
5. Or, max(Call) = S. [↑](#footnote-ref-5)
6. Exactly what “at” means depends on the specifics of the option contract. [↑](#footnote-ref-6)
7. This is a “pure” American. A “deferred” American-style option allows exercise anytime after a waiting, or deferrable, period has elapsed. [↑](#footnote-ref-7)
8. There is, of course, an arbitrage should the inequality not hold. Suppose the premium on the European – call or put – is 3.5 and on the American it is 3. Buy the American and sell the European. You have the right to exercise if you wish but the buyer to whom you sold the option may not. One of the following three events must occur:

   You hold your American option to expiration and they both finish at- or out-of-the money. You keep 0.5.

   You hold your American to expiration and they both finish in-the-money, say by 1 point. If a call, the European buyer pays you the strike. You pay the strike to the writer of the American. You pass on the security that you acquire to the buyer. If a put, the European buyer gives you the security, which you pass on to the American writer, upon which you receive the strike which you pay the buyer. In either case, you keep 0.5.

   An opportunity may present itself before expiration to exercise the American option. (We explore such opportunities in chapter qqq.) You will only do so if the profit from exercising and closing out the position exceeds the market price of the European option. You then buy the European in the market, and earn 0.5 plus that excess. [↑](#footnote-ref-8)
9. “Intrinsic” because no change is required in the price of JWN in order to produce a value of 2. [↑](#footnote-ref-9)
10. This is not necessarily an accurate statement. It will be qualified, as you will see, buy the notion of Fair Forward Value to be developed in the next section. [↑](#footnote-ref-10)
11. We looked briefly at forward contracts in chapter 1, as an example of a pure derivative. [↑](#footnote-ref-11)
12. We are making quite a number of simplifying assumptions here:

    1. No transactions costs for the cash market purchase or the contract;

    2. No storage, insurance or other costs related to holding the gold for the year;

    3. The one-year market interest rate is your borrowing rate.

    4. No initial margin above the value of the collateral required by the lender, nor marking-to-market of the

    Collateral;

    5. You face no performance risk from counterparty. [↑](#footnote-ref-12)
13. As with the long gold arbitrage earlier, we have made the simplifying assumptions of the previous footnote. The borrower may be required to pay a fee for borrowing the gold. In that case, we are effectively assuming that the interest rate earned on the cash collateral is net of this fee. Furthermore, we have implicitly assumed that the interest rate for financing is the same as the interest rate for lending the short proceeds. [↑](#footnote-ref-13)
14. Note that the equation, and its counterpart to follow, assume simple interest. As the most liquid and active options contracts are less than a year to expiry, the money market interest calculation is warranted. Otherwise, we replace the equation with FF = S x (1 + r)t. [↑](#footnote-ref-14)
15. As mentioned in the previous footnote, the interest rates in the two inequalities are not likely to be the same. This raises an important issue, as it implies that in place of the strict equality in the fair forward equation, there will be an equilibrium *range* for the fair forward. (The existence of bid-asked spreads in the spot and forward markets forces the same results.) [↑](#footnote-ref-15)
16. The two-year forward price is $1297.92 because of compounding: 1200x(1+.04)2. Here, and in the remainder of this book, we abstract from day-count conventions for calculating interest. [↑](#footnote-ref-16)
17. Actually, there is a bid and offer price for gold in the spot market. But we have enough complications to deal with without incorporating bid-offer spreads. [↑](#footnote-ref-17)
18. A static spot price is not an innocuous assumption. Indeed, as we shall see in chapter xxx, the *forward* price should be assumed to be static, and the spot price converges to *it*! [↑](#footnote-ref-18)
19. Dividends are actually paid quarterly. For simplicity, this analysis assumes the entire annual dividend occurs at the contract settlement date. Alternatively, the 0.50 represents the cumulative total of dividends including the interest earned on the payments throughout the year. [↑](#footnote-ref-19)
20. Assuming r is greater than div throughout. [↑](#footnote-ref-20)
21. Shares of utilities typically pay a relatively high dividend. Their forward prices are more likely to be at a discount to spot. [↑](#footnote-ref-21)
22. The bond’s current yield, rather than its yield-to-maturity, is the relevant carry calculation. The repurchase agreement rate is typically the cost of financing. Corporate and other non-Treasury bonds pay a risk premium above the yield on comparable maturity Treasuries. The wider the spread, the greater the net carry. [↑](#footnote-ref-22)
23. Insurance and other costs of storage magnify this relationship. Actual forward prices for other commodities – agricultural, energy, industrial metals – depart significantly from their “fair forward” values, due to the prohibitive costs of borrowing or the absence of mechanisms to do so. Indeed, in some market situations their forward prices may be below spot, a situation described as “backwardation.” [↑](#footnote-ref-23)
24. A different, but equivalent, arbitrage does not involve forward contracts. Borrow Ford for a year and sell it short. Invest the proceeds at 4%. Purchase the one-year 14-Call. At the end of the year, you will receive the 15 principal plus 0.60 in interest and be required to pay the dividend of 0.50, for net proceeds of 15.10. You must buy Ford and return the share to the lender. If Ford trades above 14 then, exercise the option. If it trades below, purchase it in the spot market. You’ll make at least 1.10, the present value of which must be the price of the call. [↑](#footnote-ref-24)
25. As long as interest rates are positive. [↑](#footnote-ref-25)
26. Parallel to the previous footnote, an equivalent arbitrage would be to borrow $15 for a year in order to purchase Ford today. Purchase the one-year 16-Put. At the end of the year, you will owe the 15 principal plus 0.60 in interest, but you will receive the dividend of 0.50, or net 15.10. If Ford trades below 16 then, exercise the option to sell at 16. If it trades above, sell it in the spot market. You have assured yourself a profit of at least 0.90.

    As long as interest rates are positive. [↑](#footnote-ref-26)
27. We’ll explore the optimality of exercising a put in this case in chapter qq. [↑](#footnote-ref-27)
28. For example, time value of options, and hedge ratios. [↑](#footnote-ref-28)
29. Equivalently, given the prices of the put and call, the price of the underlying is forced by the equation. [↑](#footnote-ref-29)
30. As mentioned in a previous chapter, a number of market factors interfere with the pure arbitrage, hence with the implications of the arbitrage assumption. These include:

    * costs of borrowing shares
    * differences between financing and lending rates
    * direct and indirect transactions costs

    The result is a *range* for the put-call relationship in place of a perfect put-call parity condition. The range will be tighter the smaller the sizes of these interferences. [↑](#footnote-ref-30)